Exercise sheet 1 - Quantum mechanics

1. The operators $A_i (i = 1, ..4)$ are defined as follows:

   (a) $A_1\psi(x) = x^3\psi(x)$

   (b) $A_2\psi(x) = \frac{d}{dx}\psi(x)$

   (c) $A_3\psi(x) = \sin[\psi(x)]$

   (d) $A_4\psi(x) = \int_a^x \psi(s)ds$

   Which of these operators are Hermitian?Which are linear? Justify your answer.

2. A quantum particle is confined in a box of length L with impenetrable walls, the centre of the wall being at the origin of the coordinates. In the ground state, the wave function of the particle is given by

$$\psi_1(x) = \begin{cases} \left(\frac{2}{L}\right)^{1/2} \cos(\pi x/L), -\frac{L}{2} < x < \frac{L}{2} \\ 0 \quad \text{otherwise} \end{cases} .$$

   (a) What is the expectation value of the position x?

   (b) What is the expectation value of the momentum of the particle?

   (c) Is the wave function an eigenfunction of the momentum operator? Justify your answer.

3. Consider the wavefunction

$$\phi(\mathbf{p}) = \frac{1}{(2\pi\hbar)^{3/2}} \int \psi(\mathbf{r}) \exp[-i\mathbf{p}.\mathbf{r}/\hbar] d^3r.$$

   (a) Show that $\hat{\mathbf{p}}\phi(\mathbf{p}) = \mathbf{p}\phi(\mathbf{p})$.Can one say this function is an eigenfunction of $\hat{\mathbf{p}}$?Why of why not?

   (b) Show that $\hat{\mathbf{r}}\phi(\mathbf{p}) = -i\hbar\nabla_p\phi(\mathbf{p})$, with $\nabla_p = \partial/\partial_{px}\mathbf{i} + \partial/\partial_{py}\mathbf{j} + \partial/\partial_{pz}\mathbf{k}$. Can one say this function is an eigenfunction of $\hat{\mathbf{r}}$?Why of why not?