PoG Exam 2011/12 Answers (without unseen section B parts)

A1. Kapteyn's universe was too small (10 kpc) while Shapley's Galactic model was too large (100 kpc) compared to the true size (50 kpc). Both models were wrong for the same reason – failure to include effects of *interstellar extinction.* [1 mark] Kapteyn looked mostly in the galactic plane (called zone of avoidance, ± 10^{o} from galactic plane) where the extinction is most severe. [1 mark] Shapley looked mostly away from the galactic plane, but since he did not account for the interstellar extinction, he overestimated distances by a factor of 2. [1 mark] The significance was in that the measured distance 500 kpc was much larger than the size of the Galaxy thus finally resolving the debate whether the observed nebulae were inside or outside our own galaxy – thus signaling the discovery of galaxies other than our own. [2 marks]. **[1+1+1+2=5 marks]**

A2.

In its rest frame, the quasar SDSS 1030+0524 produces a hydrogen emission line of wavelength $\lambda_{\rm rest} = 121.6$ nm. On Earth, this emission line is observed to have a wavelength of $\lambda_{\rm obs} = 885.2$ nm. The redshift parameter for this quasar is thus

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm rest}}{\lambda_{\rm rest}} = 6.28.$$

we may calculate the speed of recession of the quasar:

$$z = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} - 1$$

$$\frac{v_r}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

$$= 0.963.$$

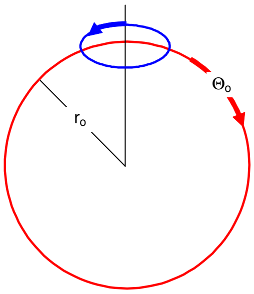
[5 marks]

A3. The surface brightness of a galaxy is the flux density (energy per area per time) per unit solid angle, I=dF/dΩ, as a function of position in the galaxian image [2 marks] Since dF=dL/(4πd^2) and dA=d^{2}dΩ, we can write this as I=(1/4π)dL/dA which is independent of d. [2 marks] **[2+2=4 marks].**

A4. Step 1: use virial theorem $2T + \Omega \approx 0$;[1 mark] Step 2: calculate potential energy of the galaxy using $\Omega = -\alpha GM^2 / R$ where alpha is constant of order unity. [1 mark] Step 3: calculate twice kinetic energy of stars using $2T \approx M\langle v^2 \rangle$ where $\langle v^2 \rangle$ is mean-square velocity. [1 mark] Step 4 combine above to get $M \approx R\langle v^2 \rangle /(\alpha G)$ and by measuring $\langle v^2 \rangle$ from Doppler broadening of an absorption line of <u>large collection of stars</u>, the mass *M* is obtained (size *R* is known from photometry measurements). [1 mark]

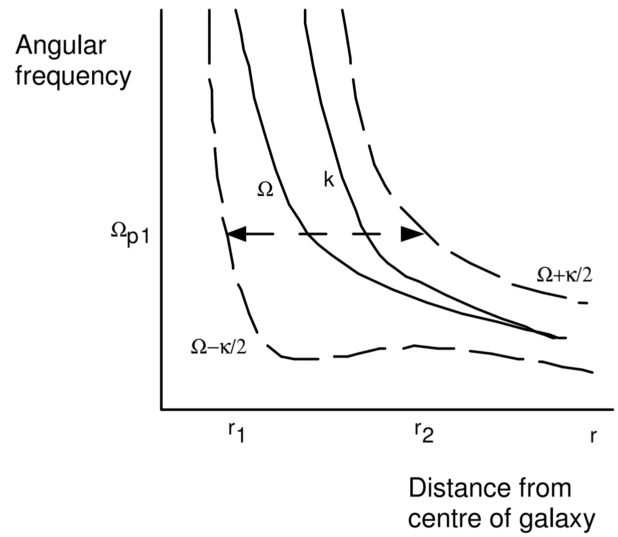
[1+1+1+1=4 marks]

A5. The star moves outward under the influence of the impulse. Because the star is given a purely radial impulse, its angular momentum is conserved. $L = mr\Theta = mr^2\Omega = mr_o^2\Omega_o = mr_o\Theta_o$. Its linear velocity about the centre of the galaxy ($\Theta = \Theta_o \frac{r_o}{r} \propto \frac{1}{r}$) therefore decreases and – because the rotation curve is flat – is therefore less than that of the stars amongst which it now finds itself [1 mark]. The stars amongst which it now finds itself have the right circular velocity to remain in circular orbit at this distance from the centre of the galaxy. The perturbed star therefore has too little and will have to drop inwards. [1 mark]



This motion is called epicyclic motion. (For the sketch [2 marks])

[1+1+2=4 marks]

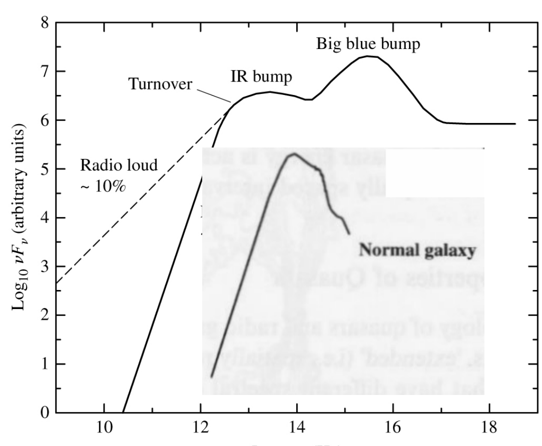


A6. *Lindblad resonances* are boundaries of the region on an angular frequency versus radius diagram at which resonances occur between the frequency with which a gas cloud meets perturbing effects of the arms (gas angular frequency Ω minus pattern angular frequency Ω_p, i.e. $\Omega - \Omega_p$) [**Note for marker: also accept $\Omega_p - \Omega$**] and its epicyclic frequency κ. It is known that resonance can have a destructive effect on a physical system or an object. (We know that, if we repeatedly "hit" something at its resonant frequency, the oscillations tend to build up and the object may destroy itself.) In the absence of any turbulent velocity, spiral density waves can therefore only exist between the Lindblad resonances, $\Omega - \kappa/m < \Omega_p < \Omega + \kappa/m$.

[4 marks]

A7. The energy must derive from conversion of mass to energy in some way. As the energies are so vast, and produced in such small regions, we should seek the most efficient means of converting stellar mass to AGN energy. Nuclear burning is only about 0.7% efficient. On the other hand conversion of mass to energy has an efficiency of ~8% (1/12 precisely). Thus the rate of consumption of stars (and so rate of replenishment if in equilibrium) is less of a problem for gravitational accretion. **[5 marks]**

A8. The spectra of AGN are unlike those of normal galaxies, having no absorption lines characteristic of starlight, but having strong emission lines [1 mark] and covering a wide range of wavelengths from the radio to X-rays/gamma rays [1 mark].



[1 mark (*marker comments: for the AGN spectrum sketch text such as "radio laud, turnover, IR bump, big blue bump" is not required – student is expected to sketch a broad, flat curve covering* *<u>all wavelength</u>, contrary to spectrum of a normal (non-active) galaxy which emits only in <u>optical part</u> of the spectrum*)]

[1+1+1=3 marks]

A9. If $k < \left(\frac{4\pi G\rho}{u^2}\right)^{1/2}$, then we have from the dispersion relation, $\omega^2 < 0$, so that ω is imaginary. Hence writing $\omega = \pm i\alpha$, where α is a real positive number and substituting into the wave-like solution for the density perturbation $\rho_1(t,x)$

$$\rho_1(t,x) = \rho_{10}e^{i(\omega t - kx)} = \rho_{10}e^{\mp\alpha t}e^{-ikx}$$

The last equation shows that we have a standing wave with amplitude $\rho_{10}e^{\mp\alpha t}$.

[5 marks]

A10. Ignoring thermal pressure support effects (zero sound speed) yields $\omega^2 = -4\pi G\rho$, which then results in exponentially growing and decaying solutions $e^{\pm i\omega t} = e^{\pm\sqrt{4\pi G\rho}t} = e^{\pm t/\tau_{ff}}$. Thus the free fall time under gravity alone (without thermal pressure support) is defined as the time over which density of the cloud increases *e* times, i.e. $\tau_{ff} = 1/\sqrt{4\pi G\rho}$ [3 marks] is the time it takes a cloud to collapse in the absence of supporting pressure [2 marks]. **[3+2=5 marks]**

A11. $\xi(r)$ measures the *excess* probability, above random, of finding a second galaxy near the first. If $\xi(r) > 0$, then the probability is greater than random, indicating that galaxies tend to cluster together. If, on the other hand, $\xi(r) < 0$, galaxies tend to avoid each other and we have anti-clustering. $\xi(r) = 0$ case is when the probability is random.

[3 marks]

A12. $\xi(r) = \left(\frac{r}{r_o}\right)^{-\gamma}$, let us put one galaxy at r=0 and another at r=10Mpc. Then the ratio of probabilities is $\frac{(1+(10/5)^{-2})}{(1+0)} = 1.25$. Thus the probability of finding another galaxy is 1.25 times larger than random.

[3 marks]

B1.

$E^2 = p^2c^2 + m^2c^4$; for massless photons

$p = E/c$. rate of change of momentum $\dot{p}$ should give us force : $\dot{p} = \dot{E}/c$ but $\dot{E} = L$ (rate of change of energy is the luminosity); radiation pressure $P_{rad} = F_{rad}/A$

$F_{rad} = \dot{p} = L/c$; $A = 4\pi r^2$; $P_{rad} = \frac{L}{c4\pi r^2}$

Force that acts in the outwards direction is

$F_{out} = P_{rad} \cdot \sigma_T$; $\sigma_T \sim \frac{1}{m^2}$ Thus outwards directed radiation pressure would acting mostly on electrons.

$F_{inwards} = \frac{GM(m_p + m_e)}{r^2} \approx \frac{GMm_p}{r^2}$ The gravity acts mostly on protons. However these are coupled by strong Electromagnetic forces so accreting plasma is quasi-neutral, i.e. inflows as a whole.

In the balance $F_{out} = F_{inwards} \Rightarrow \frac{L}{c4\pi r^2}\sigma_T = \frac{GMm_p}{r^2}$

$L = 4\pi GcMm_p/\sigma_T$

[10 marks]

$$L_{AGN} = 10^{39} W; \quad L_{AGN} = \frac{4\pi GcMm_p}{\sigma_T}; \quad \frac{M}{M_{sun}} = \frac{L_{AGN}\sigma_T}{4\pi Gcm_p M_{sun}};$$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{(1.6x10^{-19})^2}{4\pi x 8.85x10^{-12} x 9.11x10^{-31} x(3x10^8)^2}\right)^2 = 6.60x10^{-29} m^2$$

$$\frac{M}{M_{sun}} = \frac{10^{39} x 6.60x10^{-29}}{4\pi x 6.67x10^{-11} x 3x10^8 x 1.67x10^{-27} x 2x10^{30}} = 7.86x10^7$$

[3 marks]

$$r_{lso} \le c\Delta t; \quad 3\frac{2GM}{c^2} \le c\Delta t; \quad M \le \frac{c^3 \Delta t}{6G}$$

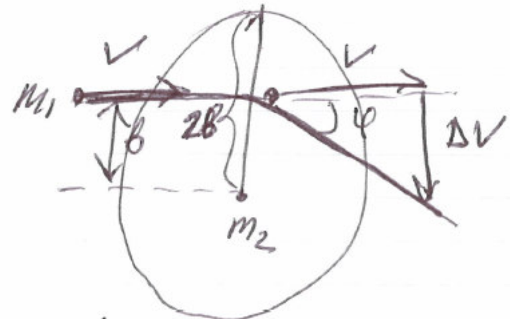
$$\frac{M}{M_{sun}} \le \frac{c^3 \Delta t}{6GM_{sun}} = \frac{(3x10^8)^3 x24x60x60}{6x6.67x10^{-11} x2x10^{30}} = 2.91x10^9$$

[3 marks]

Synchrotron radiation originates in the motion of relativistic particles in a magnetic field.

[2 marks]

B2.



$$F = \frac{Gm_1 m_2}{r^2} \quad r \to b$$

$$F = \frac{Gm_1 m_2}{b^2}$$

Impulse $I = F\Delta t$; $\Delta t = 2b/V$; $F\Delta t = m_1 \Delta V$

$$I = \frac{Gm_1 m_2}{b^2}\frac{2b}{V} = \frac{2Gm_1 m_2}{Vb} = m_1 \Delta V \Rightarrow$$

$$\Delta V = \frac{2Gm_2}{bV}$$ drop subscript "2".

$$\tan\varphi = \frac{\Delta V}{V} = \frac{2Gm}{bV^2} \quad ;$$

For N number of stars $N = V \cdot n$ where V is the total volume, n is the number density (number of stars per unit volume). Volume of influence of a star $V_{star} = \frac{V}{N} = n^{-1}$, but $V_{star} \sim b^3$

$$b^3 \approx n^{-1} \Rightarrow \quad b \approx n^{-1/3} \Rightarrow$$

$$\tan\varphi = \frac{2Gn^{1/3} m}{V^2}$$

[6+7 marks]

The assumptions are (i) that in the galactic plane the motion is nearly circular and (ii) that motion in the out-of-plane direction is independent (decoupled) from the motion in-plane direction.

[2 marks]

B3.

For a *bound* self-gravitating system the virial theorem states that the sum of twice the kinetic energy T and the potential energy Ω is zero.

Since the material is in quasi-stationary orbit about the central mass, the kinetic energy $T(r)$ and potential energy $\Omega(r)$ of material at distance r from the centre must obey the virial theorem:

$$2T(r)+\Omega(r)=0.$$

The total energy of the system – that is the sum of the kinetic energy, the potential energy and any energy $R(r)$ that has been radiated by the time the material has reached r – must be conserved, so that

$$T(r)+\Omega(r)+E(r)=0,$$

where I have taken the potential energy of the material at infinite distance from the central mass to be zero. Eliminating T, we get

$$E(r)=-\frac{1}{2}\Omega(r)=\frac{1}{2}\frac{GMm}{r};$$

$$T(r)=-\frac{1}{2}\Omega(r)=\frac{1}{2}\frac{GMm}{r}.$$

Only half the potential energy is, therefore, available to be converted into radiation.

[6 marks]

In time dt, an amount $\dot{m}dt$ falls through the annulus. Its potential energy $\Omega(r)$ at the outer edge of the annulus is given by

$$\Omega(r)=-\frac{GM\dot{m}dt}{r}.$$

So the amount $d\Omega(r)$ it loses in falling through the annulus is given by

$$d\Omega(r)=\frac{GM\dot{m}dt}{r^2}dr.$$

From the above result, therefore

$$dE(r)=\frac{1}{2}\frac{GM\dot{m}dt}{r^2}dr.$$

[5 marks]

The luminosity $dL(r)$ of the annulus of the disc is given by

$$dL(r) \equiv \frac{dE(r)}{dt} = \frac{1}{2}\frac{GM\dot{m}}{r^2}dr$$

This power is radiated by an area *dA* of the disc, given by

$$dA = 2\times(2\pi r dr),$$

where the extra factor of two appears because the disc has two sides. Let the luminosity *per unit area* of the disc at radius *r* be *l*(*r*). Then

$$2\times(2\pi r dr)\times l(r) = \frac{1}{2}\frac{GM\dot{m}}{r^2}dr$$

and (neglecting the viscosity in the disc)

$$F(r) = \frac{GM\dot{m}}{8\pi r^3}.$$ **[6 marks]**

Assuming that the disc radiates as a black body, we have

$$l(r) = \sigma T^4(r)$$

so that

$$T(r) = \left[\frac{l(r)}{\sigma}\right]^{1/4} = \left(\frac{GM\dot{m}}{8\pi r^3\sigma}\right)^{1/4}.$$ **[3 marks]**

B4.

a) If $k < \left(\frac{4\pi G\rho}{u^2}\right)^{1/2}$, then we have from the dispersion relation, $\omega^2 < 0$, so that ω is imaginary which leads to the Jeans instability i.e. exponentially growing solutions. This occurs for the case of Jeans length of $\lambda > \lambda_J = \left(\frac{\pi u^2}{G\rho}\right)^{1/2}$

[4 marks].

b) Jeans mass can be obtained from $M = (4\pi/3)R^3\rho$ [1 mark]. If the diameter of the collapsing cloud is *l*=2*R*, one needs to realize that for the possible modes of oscillation, cloud edges need to be nodes of the standing wave because outside the cloud no oscillation can be sustained [1 mark]. A trough or crest of the wave (bounded by cloud edges) is then half the wavelength. Therefore, $l = \lambda_J/2$ and $R = \lambda_J/4$ [2 marks]. Thus,

$$M_J = (4\pi/3)(\lambda_J/4)^3\rho = \frac{4\pi}{3\cdot 4^3}\left(\frac{\pi u^2}{G\rho}\right)^{3/2}\rho = \frac{\pi^{5/2}}{48}\frac{u^3}{G^{3/2}}\frac{1}{\sqrt{\rho}}$$ [2 marks]. **[1+1+2+2=6 marks].**

c)

i) The result of linearization of the equations ($f = f_0 + \varepsilon f_1 + ...$), assuming that unperturbed state has properties $\rho_0 = const,\ \ V_0 = 0,\ \ p_0 = const,\ \ \varphi_0 = const$, by collecting terms at order ε is: (i.e. O(ε))

$$\varepsilon \frac{\partial \rho_1}{\partial t} + \varepsilon \nabla \cdot (\rho_0 \vec{V}_1) = \varepsilon \frac{\partial \rho_1}{\partial t} + \varepsilon \rho_0 \nabla \cdot (\vec{V}_1) = 0,$$

$$\varepsilon \frac{\partial \vec{V}_1}{\partial t} = -\varepsilon \frac{\nabla p_1}{\rho_0} - \varepsilon \nabla \varphi_1,$$

$$\varepsilon \Delta \varphi_1 = \varepsilon 4\pi G \rho_1,$$

applying ∇ both sides for substitution into above $\nabla p = const\ \gamma\ \rho^{\gamma-1} \nabla \rho = \frac{\gamma\, const \rho^{\gamma}}{\rho} \nabla \rho = \frac{\gamma p}{\rho} \nabla \rho = u^2 \nabla \rho,$

$$\nabla p_1 = u^2 \nabla \rho_1$$

[5 marks]