Queen Mary
University of London

BSc/MSci Examination

Date 31May 2012 Time 10:00-12:30

PHY305 Physics of Galaxies Duration: 2 hours 30 minutes

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Candidates should answer ALL questions in Section A and TWO of the four questions in Section B. An indicative marking-scheme is shown in square brackets [ ] after each part of a question.

CALCULATORS ARE PERMITTED IN THIS EXAMINATION. PLEASE STATE ON YOUR ANSWER BOOK THE NAME AND TYPE OF MACHINE USED.

COMPLETE ALL ROUGH WORKINGS IN THE ANSWER BOOK AND CROSS THROUGH ANY WORK WHICH IS NOT TO BE ASSESSED.

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EXAM PAPERS CANNOT BE REMOVED FROM THE EXAM ROOM

Examiners: Dr D Tsiklauri (Module Organiser) and Dr TJS Dennis (Deputy Module Organiser)

Appendix on the last page follows

Section A (*answer ALL questions in Section A*) **[50 Marks in Total]**

Question A1

Kapteyn's model of the Galaxy was much smaller and thicker, relative to its diameter, than currently accepted models; whereas Shapley's model was much larger and thinner. Briefly comment on the reasons for the above two discrepancies. What was the significance of measuring distance to the Andromeda galaxy for the first time by Edwin Hubble?

[5 marks]

Question A2

In its rest frame, the quasar SDSS 1030+0524 produces a hydrogen emission line of wavelength $\lambda_{rest} = 121.6$ nm. On Earth, this emission line is observed to have a wavelength of $\lambda_{obs} = 885.2$ nm. The redshift parameter for this quasar is thus $z = (\lambda_{obs} - \lambda_{rest}) / \lambda_{rest} = 6.28$. Calculate line-of-sight recession velocity of the quasar (avoiding the erroneous temptation to answer that it moves 6.28 times faster than speed of light!). In general, if one has an electromagnetic wave propagating away from an observer, Doppler shift reduces its frequency according to $f_{obs} = f_{rest}\sqrt{(1-v/c)/(1+v/c)}$.

[5 marks]

Question A3

Give the definition of the surface brightness $I(\theta)$ of a galaxy and show that it is independent of the distance to the galaxy.

[4 marks]

Question A4

Describe briefly and qualitatively how the virial theorem is used to estimate the masses of elliptical galaxies.

[4 marks]

Question A5

Describe with an aid of a sketch and qualitative physical explanation why stars move on *epicyclic* orbits in a spiral galaxy.

[4 marks]

Question A6

Sketch and state physical meaning of the *Lindblad resonances*, $\Omega - \kappa / m < \Omega_p < \Omega + \kappa / m$, where symbols have their usual meaning.

[4 marks]

Question A7

Briefly explain why gravitational accretion is more likely to power active galactic nuclei (AGN) than thermonuclear fusion, by comparing the efficiencies of conversion of mass to energy in both cases.

[5 marks]

Question A8

Explain how the spectrum of active galaxies is different from that of normal ones. Include typical sketch (both on the same plot) of the spectra for the two types of galaxies.

[3 marks]

Question A9

Using the dispersion relation for small perturbations of a self-gravitating gas with non-zero pressure, $\omega^2 = u^2k^2 - 4\pi G\rho$, show that a perturbation with wave-number k given by $k < \left(4\pi G\rho / u^2\right)^{1/2}$ is a *standing wave*, with exponentially growing or decaying amplitude.

[5 marks]

Question A10

Use the dispersion relation from question A9 above, with the thermal pressure effects neglected, to derive *the free fall time* and explain its physical meaning.

[5 marks]

Question A11

The probability of finding a galaxy within volume dV_1 at a location $\mathbf{r}_1$ and another galaxy within dV_2 at $\mathbf{r}_2$ is given by $P(\mathbf{r}_1,\mathbf{r}_2)dV_1dV_2$. The number of galaxies per unit volume is φ. If the distribution of galaxies is isotropic but not random $P(\mathbf{r}_1,\mathbf{r}_2) = \varphi(1+\xi(\mathrm{r}))$. What is the physical meaning of the *two point correlation function* $\xi(\mathrm{r})$? Comment on the cases when $\xi(\mathrm{r})$ is positive, negative or zero.

[3 marks]

Question A12

A survey of galaxies has established that $\xi(r) = (r / r_0)^{-\gamma}$ with r_0=5 Mpc and $\gamma = 2$. Compare the probability of finding a galaxy 10 Mpc away from another one in the survey to that of random distribution of galaxies.

[3 marks]

Section B (*answer TWO of the four questions in Section B*)

Question B1 **[25 Marks in Total]**

(a) Consider accretion of material into an AGN. Show that the accretion stops when the luminosity exceeds the Eddington limit $L_E = 4\pi GcM_{BH} m_p / \sigma_T$. σ_T is the Thomson cross-section (cross-section of scattering of a photon with a massive, charged particle); $\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 mc^2}\right)^2$, where m is a charged particle's mass. The relativistic relation $E^2 = p^2c^2 + m^2c^4$ applies. Explain the relative importance of the radiation and gravity forces on protons and electrons; and state why these particles accrete as a single fluid.

[10 marks]

(b) If an AGN emits at the Eddington luminosity limit of 10^{39} W, what is the mass (in solar masses) of the central BH?

[3 marks]

(c) Assume that size of the emitting region on the AGN is roughly three Schwarzschild radii. Estimate the mass of the BH, in solar masses, if the AGN varies on the time scale of 24 hours.

[3 marks]

(d) What is the origin of *synchrotron radiation*?

[2 marks]

(e) Over a wide range of frequencies the radio continuum spectrum of many radio galaxies can be approximated by a power law $S_\nu \propto \nu^\alpha$ where S_ν is the flux density at radiation frequency ν and $\alpha \approx$ -0.75. Show that this is consistent with the observed emission being synchrotron radiation from a population of relativistic electrons with a power law energy distribution

$$N(E)dE \propto E^{-p} dE$$

where $p \approx 2.5$. [Hint: An energetic electron with energy E in the presence of a magnetic field B radiates a power $P \propto B^2E^2$ at a frequency $\nu \propto E^2B$.]

[7 marks]

Question B2 **[25 Marks in Total]**

(a) In a galaxy, two gravitationally interacting stars have an impact parameter b and relative speed v.

(i) Prove that velocity change Δv due to the mutual gravitational interaction of the stars is $\Delta v = 2Gm/(bv)$, where m is mass of the star that causes the orbit deflection. [Hint: you may assume that the distance between the stars on average is b and the range (length) of the gravitational interaction, as one star passes by the other, is roughly $2b$].

[6 marks]

(ii) Show that tangent of the deflection angle of stars via their mutual gravitational interaction in a galaxy is given by

$$\tan\phi = 2Gmn^{1/3} / v^2$$

where, n is number density of stars. [Hint: Estimate star's volume of "influence" to relate n and b.]

[7 marks]

(b) State the two main assumptions made when describing the dynamics of stars in spiral galaxies.

[2 marks]

(c) The dynamics of stars (e.g. that of the Sun) in spiral galaxies in the out-of-galactic plane (z) direction is described by the equation

$$\ddot{z} = -(4\pi G\rho)z.$$

Marine life fossil record indicates that the life extinction occurs with 64 Myr periodicity. Calculate the density ρ in the solar neighbourhood in M_{sun} pc^{-3}. Does your answer for ρ include the effect of the presence of dark matter? Justify your answer.

[10 marks]

Question B3 **[25 Marks in Total]**

(a) Use the virial theorem along with the conservation of energy to show that, in an accretion disc, half the loss of potential energy of the accreting material is converted into radiation.

[6 marks]

(b) The potential energy $\Omega(r)$ of a mass m at distance r from a black hole of mass M is

$$\Omega(r) = -\frac{GMm}{r}.$$

(i) Deduce that the energy $dE(r)$ radiated in time dt by an annulus of width dr in an accretion disc around a black hole is given by

$$dE(r) = \frac{1}{2}\frac{GM\dot{m}dt}{r^2}dr\,,$$

where $\dot{m}$ is the accretion rate.

[5 marks]

(ii) Show that the flux $F(r)$ (i.e. luminosity per unit surface area) of the disc is

$$F(r) \approx \frac{GM\dot{m}}{8\pi r^3}\quad .$$

[6 marks]

(c) (i) Assuming that the disc radiates as a black body, show that its temperature at radius r is given approximately by

$$T(r) = \left(\frac{GM\dot{m}}{8\pi r^3 \sigma}\right)^{1/4}\quad .$$

[3 marks]

(ii) Assuming that the radius of the last stable circular orbit around such a black hole is three times the Schwarzschild radius, show that the maximum temperature, $T_{\max}$, of the accretion disc is

$$T_{\max} = \left(\frac{\dot{m}}{12\pi\sigma}\right)^{1/4}\left(\frac{c^3}{12GM}\right)^{1/2}\quad .$$

[3 marks]

(iii) Calculate this maximum temperature if the black hole with mass M=$10^8 M_{sun}$ is accreting material at the rate of one solar mass a year.

[2 marks]

Question B4 **[25 Marks in Total]**

a) Use the dispersion relation $\omega^2 = u^2k^2 - 4\pi G\rho$ to derive *Jeans length*, $\lambda_J = 2\pi / k_J = \left(\pi u^2 / G\rho\right)^{1/2}$.

[4 marks]

b) Derive *Jeans mass*, $M_J = \dfrac{\pi^{5/2}}{48}\dfrac{u^3}{G^{3/2}}\dfrac{1}{\sqrt{\rho}}$.

[6 marks]

c) Use the hydrodynamic equations

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0,$$
$$\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V} = -\frac{\nabla p}{\rho} - \nabla\varphi,$$
$$\Delta\varphi = 4\pi G\rho,$$
$$p = const\rho^{\gamma},$$

where the symbols have their usual meaning, to derive the dispersion relation for small perturbations of a self-gravitating gas with non-zero pressure:

$$\omega^2 = u^2k^2 - 4\pi G\rho.$$

Here ω is the angular frequency, k is the wave-number, $u = \sqrt{\gamma p / \rho}$ is the sound speed and ρ is the unperturbed gas density. The derivation should clearly show each of the following steps:

(i) linearization of the equations ($f = f_0 + \varepsilon f_1 + \ldots$), assuming that unperturbed state has the following properties: $\rho_0 = const$, $V_0 = 0$, $p_0 = const$, $\varphi_0 = const$,

[5 marks]

(ii) Fourier ansatz ($f = \tilde{f}e^{i(\omega t - \vec{k}\cdot\vec{r})}$),

[5 marks]

(iii) use of the compatibility condition of the linear algebraic equations (determinant of the main matrix should be zero).

[5marks]

End of Examination. An appendix on one page follows.

Appendix

List of physical constants

Velocity of light	c	3.00×10^{8}	m s^{-1}
Gravitational constant	G	6.67×10^{-11}	N m^{2} kg^{-2}
Boltzmann constant	k	1.38×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8}	W m^{-2} K^{-4}
Planck constant	h	6.63×10^{-34}	J s
Mass of proton	m_p	1.67×10^{-27}	kg
Mass of electron	m_e	9.11×10^{-31}	kg
Electron charge	e	1.60×10^{-19}	C
vacuum permittivity	ε_0	8.85×10^{-12}	F m^{-1}
vacuum permeability	μ_0	$4\pi \times 10^{-7}$	H m^{-1}
Mass of Sun	M_{sun}	1.99×10^{30}	kg
Luminosity of Sun	L_{sun}	3.83×10^{26}	W
Astronomical unit	AU	1.50×10^{11}	m
Parsec	pc	3.09×10^{16}	m
Year	yr	3.16×10^{7}	s