Physics of Galaxies Exercises: Set 6

1. For a perfect gas, $u = p/(\gamma-1)$ where $\gamma = c_p/c_v$ is the ratio of specific heats. Deduce that the density change across the shock can be expressed as

$$\frac{\rho_f}{\rho_i} = \left[\frac{\frac{(\gamma_f+1)}{(\gamma_f-1)} p_f + p_i}{p_f + \frac{(\gamma_i+1)}{(\gamma_i-1)} p_i} \right] .$$ **[5 marks]**

[This fills in some of the algebra in the lectures.]

2. Explain **briefly** and **qualitatively** why the motion of stars perpendicular to the disc of a spiral galaxy is almost entirely independent of the motion within the disc **[4 marks].** Now use **quantitative** argument to prove the same, using the facts that (i) a spiral galaxy's vertical scale-height is 1 kpc and (ii) typical random velocity of a star is 20 km/s **[3 marks]**. In your estimate, why would you not use the bulk rotational velocity around the galactic centre (which is typically 200 km/s)? **[1 mark]**

3. Show that, for a galaxy with a flat rotation curve, the angular velocity of stars in circular orbit about the centre of the galaxy is inversely proportional to the distance from the centre. **[3 marks]**

4. The Oort parameters A(r) and B(r) are given, as a function of the distance *r* from the centre of a disc galaxy, by

$$A(r) = +\frac{1}{2}\left[\frac{\Theta(r)}{r} - \frac{d\Theta(r)}{dr}\right] \quad B(r) = -\frac{1}{2}\left[\frac{\Theta(r)}{r} + \frac{d\Theta(r)}{dr}\right].$$

At the distance r_o of the of the Sun from the centre of the Galaxy, the Oort parameters have the values

$A_o := A(r_o) = +14.4 \pm 1.2$ km s^{-1} kpc^{-1};

$B_o := B(r_o) = -12.0 \pm 2.8$ km s^{-1} kpc^{-1};

$r_o = 8.5 \pm 1.0$ kpc.

Neglecting the quoted uncertainties, calculate the circular velocity Θ_o in km s^{-1} **[2 mark]** and the period of rotation T_o in years **[2 marks]** of the Sun about the centre of the Galaxy.

Show that, in the outer regions the disc, where the circular velocity $\Theta(r)$ is given by $\Theta(r) \approx \Theta_o = \text{constant}$, $A(r)$ and $B(r)$ are given by

$$A(r) \approx -B(r) \approx \frac{1}{2}\frac{\Theta_o}{r} .$$ **[2 marks]**

22 marks in total