Physics of Galaxies Exercises: Set 5

1. Prove that velocity change Δv due to the mutual gravitational interaction of the stars is $\Delta v = 2Gm/(bv)$, where *m* is mass of the star that causes the orbit deflection. [Hint: you may assume that the distance between the stars on average is *b* and the range (length) of the gravitational interaction, as one star passes by the other, is roughly 2*b*]. **[5 marks]**

2. Show that tangent of the deflection angle of stars via their mutual gravitational interaction in a galaxy is given by $\tan\phi = 2Gmn^{1/3}/v^2$ where, *n* is number density of stars. [Hint: Estimate star's volume of "influence" to relate *n* and *b*.] **[5 marks]**

3. The sound speed, *u*, for a gas of density ρ and pressure *p* is also defined by

$$u^2 = \left[\frac{\partial p}{\partial \rho}\right]_{\text{adiabatic}}.$$

For a perfect gas behaving adiabatically, we have

$$p\rho^{-\gamma} = \text{constant},$$

where γ is the ratio of specific heats. Show that

$$u \propto \rho^{(\gamma-1)/2}.$$ **[3 marks]**

4. The free-fall time τ_{ff} of an interstellar cloud of initial density ρ is given by

$$\tau_{\text{ff}} \sim (G\rho)^{-1/2}.$$

Estimate the free-fall time of a cloud with an initial number-density of 10^{10} hydrogen molecules per cubic metre. **[3 marks]**

5. When self-gravitation is taken into account, the dispersion relation for sound waves in a cloud of unperturbed density ρ is

$$\omega^2 = u^2k^2 - 4\pi G\rho,$$

where ω is the angular frequency and *k* is the wave-number. Show that a perturbation with wave-number *k* given by

$$k < \left(\frac{4\pi G\rho}{u^2}\right)^{1/2}$$

is a *standing wave*, with an exponentially growing or decaying amplitude. **[5 marks]**

21 marks in total