**Physics of Galaxies Exercises: Set 4**

1. Look up Salpeter initial mass function expression in the lecture notes and derive the relation for the total mass in the Galaxy as

$$M_{tot} = \frac{\phi_* M_*^2}{\alpha - 2}\left(\frac{M_l}{M_*}\right)^{2-\alpha} .$$

Show each step of the calculation explicitly **[4 marks]**. Explain, without invoking mathematics, why the total mass of the galaxy is prescribed by low mass stars with mass $M_l$, whereas the total luminosity is prescribed by high mass stars with mass $M_h$. **[2 marks]**.

2. Distances of galaxies are obtained by measuring their redshifts $z$, calculating their recession velocities $v_H$ and using Hubble's law to get the distance $d$:

$$d = v_H / H_o = zc / H_o \quad .$$

Show that the luminosity $L$ of a galaxy is given in terms of its *measured* flux-density $F$ by

$$L = 4\pi\left(z\frac{c}{H_o}\right)^2 F \; . \qquad \textbf{[4 marks]}$$

Masses of galaxies are derived from an expression of the form

$$M = kRv^2 / G ,$$

where $R$ is radius of the galaxy, v is characteristic velocity of the material in it, and $k$ is a constant. Deduce that

$$M = \left(z\frac{c}{H_o}\right)\frac{v^2}{G}\theta ,$$

where $\theta$ is the *measured* angular size corresponding to the linear size $R$. **[3 marks]**
Hence show that the *measured* mass-luminosity ratio $M/L$ of galaxies is directly proportional to the Hubble constant:

$$M / L \propto H_o \propto h \qquad \textbf{[4 marks]}$$

3. Explain briefly and qualitatively why most stars in galaxies can be considered to move independently of other individual stars. [**5 marks]** What is meant by a *strong gravitational interaction*? **[2 marks]**

4. For stars of mass $m$, number density $n$ and speed $V$, the deflection experienced in a typical encounter was shown in the lectures to be

$$\tan\phi = \frac{2Gmn^{1/3}}{V^2} \approx \phi \; .$$

In the local (solar) neighbourhood, one can assume $m$ = 1 $M_{sun}$, $n$ = 0.1 $pc^{-3}$ and $V$ = 40 $kms^{-1}$. Calculate $\varphi$ in arcseconds. **[3 marks]**

**27 marks in total**