Physics of Galaxies Exercises: Set 3

1. Define what is meant by the surface brightness $I(\theta)$ of a galaxy. **[3 marks]**
The surface brightness of elliptical galaxies, and of the nuclear regions of many spiral galaxies, is well described by the de Vaucouleurs profile:

$$I(\theta) = I(0)\exp\left[-\left(\frac{\theta}{\theta_o}\right)^{1/4}\right].$$

Sketch this function labelling *x* and *y* axis properly, including for $\theta = \theta_o$. **[3 marks]**
Show that the total flux-density *F* emitted by a galaxy with surface brightness $I(\theta)$ is given by

$$F = \int_0^\infty I(\theta) 2\pi\theta d\theta. \qquad \textbf{[3 marks]}$$

Show that, for a de Vaucouleurs profile, *F* is given by

$$F = 8!\pi\theta_o^2 I(0) \qquad \textbf{[5 marks]}$$

[You may use without proof the following standard integral: $\int_0^\infty x^n e^{-x} dx = n!$]

2. It can be shown that de Vaucouleurs profile $I(\theta) = I(0)\exp\left[-\left(\frac{\theta}{\theta_o}\right)^{1/4}\right]$ can be rewritten as $I(\theta) = I(0)\exp\left[-7.67\left(\frac{\theta}{\theta_e}\right)^{1/4}\right]$, where θ_e is so-called *half-light angle* such that the light emitted within circle with radius θ_e is half of the total light, in other words

$$\int_0^{\theta_e} I(\theta) 2\pi\theta d\theta = \frac{1}{2}\int_0^\infty I(\theta) 2\pi\theta d\theta \qquad \text{and} \qquad \text{also } I_e = I(\theta_e).$$

Show that $I(\theta) = I_e \exp\left[-7.67\left[\left(\frac{\theta}{\theta_e}\right)^{1/4} - 1\right]\right]$ **[6 marks]**

20 marks in total