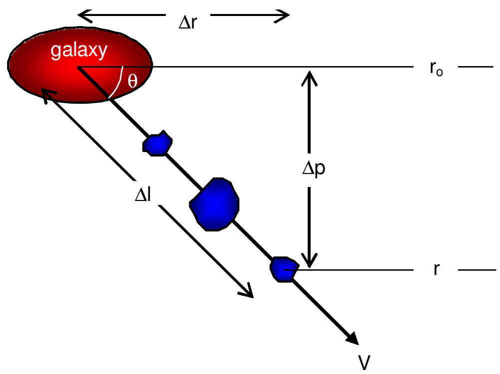
Physics of Galaxies

ANSWERS: SET NUMBER 9

1.

The diagram from Chapter 4 of Notes:



The observer will be seeing radiation from a cloud emitted (or lit up by a moving beam of electrons) near the galaxy. This radiation travels distance r_0 to observer at the earth, taking a time r_0/c. At a later time the cloud has moved with speed V to the furthest position shown in the figure, a distance Δl taking time $\Delta l/V$ so that the light received at earth has now only to travel a distance $r < r_0$ in a time r/c, giving a total time to arrive at earth $r/c+\Delta l/V$. To observers on earth the cloud would *appear* to have travelled a distance Δp in the plane of the sky at speed $\dot{p}$ which in some cases is greater than c. However the observed difference in arrival times at earth between emission near the galaxy and furthest out is $\Delta t_{obs}= (r/c+\Delta l/V)- r_0/c = \Delta l/V - \Delta r/c$ (using $\Delta r = r_0 - r$). The key point is that for a given $\Delta l/V$ this time can be made less by making Δr *greater*; and this is accomplished by making the angle θ *smaller*, as we can see from the diagram. The velocity V required to explain these observations is < c provided the angle θ is small enough and the velocity V relativistic (but still <c). (*I did not ask for a full derivation of the result, but the students may provide one! (for which The object of the exercise was to see the key point qualitatively.*) The expression relating the observed to the actual velocity of the cloud is

$$\beta_{obs} = \frac{\dot{p}}{c} = \frac{\beta \sin\theta}{1-\beta\cos\theta}$$

where $\beta = V/c$ and $\beta_{\rm obs} = \dot{p}/c$. **[7 marks]**

Now solve for β:

$$\beta = \frac{\beta_{obs}}{(\sin\theta + \beta_{obs}\cos\theta)}$$

To find the minimum β differentiate with respect to θ and equate to zero:

$$\frac{d\beta}{d\theta} = \frac{-\beta_{obs}(\cos\theta - \beta_{obs}\sin\theta)}{(\sin\theta + \beta_{obs}\cos\theta)^2} = 0 \text{, whence}$$

$$(\cos\theta - \beta_{obs}\sin\theta) = 0 \quad \text{i.e.} \quad \tan\theta = \frac{1}{\beta_{obs}}$$

substituting back into equation and using the trig identities

$$\cos^2\theta = \frac{1}{1+\tan^2\theta} = \frac{\beta_{obs}^2}{1+\beta_{obs}^2} \,.\, \sin^2\theta = \frac{\tan^2\theta}{1+\tan^2\theta} = \frac{1}{1+\beta_{obs}^2}$$

then gives the minimum β:

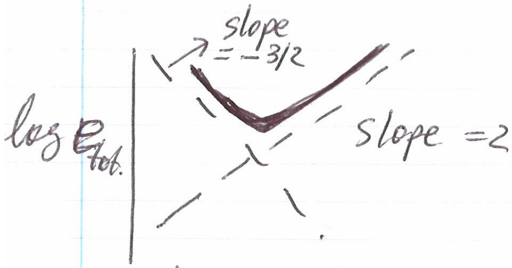
$$\beta_{\min} = \frac{\beta_{obs}}{(1+\beta_{obs}^2)^{1/2}}$$ **[4 marks]**

With $\beta_{obs} = 10$ we have

$$\beta_{\min} = \frac{10}{(101)^{1/2}} = 0.995$$

and the corresponding angle,

$\theta = 5.7^\circ$ **[3 marks]**



minumum e_{total} is obtained when $\frac{de_{total}}{dB} = 0$

$$\frac{2B}{2\mu_0} + \frac{(1+K)}{V}\frac{\sqrt{a}}{b} L \nu_0^{-1/2}\left(-\frac{3}{2}\right)B^{-5/2} = 0$$

$$\frac{1}{\mu_0} = \frac{3}{2}\frac{(1+K)}{V}\frac{\sqrt{a}}{b} L \nu_0^{-1/2} B^{-7/2} \; ; \quad B_{min} = \left[\frac{3}{2}\frac{\mu_0}{V}\frac{\sqrt{a}}{b} L \nu_0^{-1/2}(1+K)\right]^{2/7}$$

$$\frac{e_{particles}}{e_{field}} = \frac{\frac{(1+K)}{V}\frac{\sqrt{a}}{b} L \nu_0^{-1/2} B_{min}^{-3/2}}{\frac{B_{min}^2}{2\mu_0}} = 2\mu_0 \frac{(1+K)}{V}\frac{\sqrt{a}}{b} L \nu_0^{-1/2} B_{min}^{-7/2}$$

$$\frac{e_{particles}}{e_{field}} = \frac{2\cancel{\mu_0}\cancel{(1+K)}}{\cancel{V}}\frac{\cancel{\sqrt{a}}}{\cancel{b}} \cancel{L \nu_0^{-1/2}} \frac{2}{3}\frac{\cancel{V}}{\cancel{\mu_0}}\frac{\cancel{b}}{\cancel{\sqrt{a}}}\frac{1}{\cancel{(1+K) L \nu_0^{-1/2}}} = \frac{4}{3} \approx 1$$

[10 marks]

[TOTAL MARKS AVAILABLE 24]