Physics of Galaxies

ANSWERS: SET NUMBER 6

1. Since $\Omega(r)=\Theta(r)/r$, this is roughly constant if $\Theta(r)\propto r$ applies, so we have $\Omega(r)=\Omega_o=\text{constant}$. **[2 marks]**

From equations of the question, we then have

$$A(r)=\frac{1}{2}\left\{\frac{r\Omega_o}{r}-\frac{d}{dr}\left[r\Omega_o\right]\right\}=0$$,

while

$$B(r)=-\frac{1}{2}\left\{\frac{r\Omega_o}{r}+\frac{d}{dr}\left[r\Omega_o\right]\right\}=-\Omega_o$$ **[3 marks]**

From equation of the question and equations and , we have in the inner regions,

$$\kappa^2(r)=-4\left(-\Omega_o\right)\left[0+\Omega_o\right]=4\Omega_o^2$$

so that

$$\kappa(r)=2\Omega_o=\text{constant}$$. **[2 marks]**

In the outer regions we know that rotation curves are flat and $\Theta(r)=\Theta_o=$ constant. From equations (1.1) of the question and equation (1.3), we therefore have

$$\kappa^2=-4\left(-\frac{1}{2}\frac{\Theta_o}{r}\right)\left[\frac{1}{2}\frac{\Theta_o}{r}+\frac{1}{2}\frac{\Theta_o}{r}\right]=2\left(\frac{\Theta_o}{r}\right)^2$$

so that

$$\kappa(r)=\sqrt{2}\frac{\Theta_o}{r}$$. **[3 marks]**

2. The condition for closure of orbits in an inertial frame are that

$$\frac{\kappa(r)}{\Omega(r)}=\frac{q}{p}$$.

where *p* and *q* are integers. For the inner regions, we have from equation ,

$$\frac{\kappa(r)}{\Omega(r)}=\frac{2\Omega_o}{\Omega_o}=2$$.

The epicyclic orbits *are* therefore closed. In the outer regions, however, we have from equation

$$\frac{\kappa(r)}{\Omega(r)}=\frac{\kappa(r)}{\Theta_o/r}=\left(\sqrt{2}\frac{\Theta_o}{r}\right)/\left(\frac{\Theta_o}{r}\right)=\sqrt{2}$$,

which is irrational. The epicyclic orbits *are* therefore *not* closed. **[3 marks]**

3. From equation of the question and equation , we find that the resonances are at

$$\Omega(r)\pm\frac{\sqrt{2}\Omega(r)}{2}=\Omega(r)\left[1\pm\frac{1}{\sqrt{2}}\right]=\frac{\Theta_o}{r}\left[1\pm\frac{1}{\sqrt{2}}\right]$$.

giving

$$\Omega_{\text{inner}}(r)=\frac{\Theta_o}{r}\left[1-\frac{1}{\sqrt{2}}\right]=0.3\frac{\Theta_o}{r}$$ and

$$\Omega_{\text{outer}}(r)=\frac{\Theta_o}{r}\left[1+\frac{1}{\sqrt{2}}\right]=1.7\frac{\Theta_o}{r}$$. **[4 marks]**