PoG Exercise 9 Answers

1.

$$-\frac{dE(t)}{dt} = P(E) = \beta B^2 E^2 \; ; \quad dE(t) = -\beta B^2 E^2 dt$$

$$-\frac{dE}{E^2} = \beta B^2 dt \; ; \quad \int_{E_0}^{E} d\left(\frac{1}{E}\right) = \int_0^t \beta B^2 dt$$

$$\frac{1}{E} - \frac{1}{E_0} = \beta B^2 t \; ; \quad \frac{E_0}{E} = 1 + \beta B^2 E_0 t$$

$$E = E_0 / (1 + \beta B^2 E_0 t);$$

The time it takes to reduce the energy to half of its original value $E = E_0/2$ is the characteristic time of electrons losing their energy.

$$\frac{E_0}{2} = \frac{E_0}{1 + \beta B^2 E_0 \tau} \; , \quad 2 = 1 + \beta B^2 E_0 \tau \; ;$$

$$\tau(E) = 1/\beta B^2 E \; , \text{ recall that } E = \left(\frac{\nu}{a B}\right)^{1/2}$$

$$\tau(E) = \frac{1}{\beta B^2} \frac{a^{1/2} B^{1/2}}{\nu^{1/2}} = \frac{a^{1/2}}{\beta} B^{-3/2} \nu^{-1/2}$$

[8 marks]

2.

Synchrotron radiation originates in the motion of relativistic particles in a magnetic field. **[2 marks]**

Since $\nu \sim E^2 B$, we have $E \sim (\nu/B)^{1/2}$ and $d\nu \sim 2\,EB\,dE$.

The flux in the frequency range $(\nu, \nu+d\nu)$ is therefore

$S_\nu\, d\nu \sim N(E)\, P(E)\, dE \sim E^{-p} B^2 E^2 E^{-1} B^{-1}\, d\nu \sim E^{1-p} B\, d\nu$

Substituting for E, we get $S_\nu \sim (\nu/B)^{(1-p)/2} B \sim \nu^{(1-p)/2}$

where we can neglect B dependence. Putting p=2.5 gives $\alpha = -0.75$ as observed. **[8 marks]**

3.

$$L_{AGN} = 10^{39} W; \quad L_{AGN} = \frac{4\pi GcMm_p}{\sigma_T}; \quad \frac{M}{M_{sun}} = \frac{L_{AGN}\sigma_T}{4\pi Gcm_p M_{sun}};$$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{(1.6x10^{-19})^2}{4\pi x8.85x10^{-12} x9.11x10^{-31} x(3x10^8)^2}\right)^2 = 6.60x10^{-29} m^2$$

$$\frac{M}{M_{sun}} = \frac{10^{39} x6.60x10^{-29}}{4\pi x6.67x10^{-11} x3x10^8 x1.67x10^{-27} x2x10^{30}} = 7.86x10^7$$

[3 marks]

4.

$$r_{lso} \le c\Delta t; \quad 3\frac{2GM}{c^2} \le c\Delta t; \quad M \le \frac{c^3\Delta t}{6G}$$

$$\frac{M}{M_{sun}} \le \frac{c^3\Delta t}{6GM_{sun}} = \frac{(3x10^8)^3 x24x60x60}{6x6.67x10^{-11} x2x10^{30}} = 2.91x10^9$$

[3 marks]