Physics of Galaxies

ANSWERS: EXERCISE NUMBER 6

1. [Note for marker: there are different ways to obtain the answer. Full marks for all logically correct derivations should be given.]

The energy conservation equation gives -- i.e. we take this as given:

$$\left(\frac{p_f+u_f}{\rho_f}\right)-\left(\frac{p_i+u_i}{\rho_i}\right)=\frac{1}{2}(V_S^2-V_f^2)=\frac{1}{2}\left(\frac{\rho_i+\rho_f}{\rho_i\rho_f}\right)(p_f-p_i).$$

Then we can rewrite this as

$$\frac{p_f+u_f}{\rho_f}-\frac{1}{2}\frac{\rho_i+\rho_f}{\rho_i}\frac{p_f}{\rho_f}=\frac{p_i+u_i}{\rho_i}-\frac{1}{2}\frac{\rho_i+\rho_f}{\rho_f}\frac{p_i}{\rho_i}.$$

Since $u=p/(\gamma-1)$, the left-hand-side becomes

$$\left(\frac{\gamma_f}{\gamma_f-1}\right)\left(\frac{p_f}{\rho_f}\right)-\frac{1}{2}\left(\frac{p_f}{\rho_f}\right)-\frac{1}{2}\left(\frac{p_f}{\rho_i}\right)=\frac{1}{2}\left(\frac{\gamma_f+1}{\gamma_f-1}\right)\left(\frac{p_f}{\rho_f}\right)-\frac{1}{2}\left(\frac{p_f}{\rho_i}\right).$$

Similarly the right-hand-side becomes

$$\frac{1}{2}\left(\frac{\gamma_i+1}{\gamma_i-1}\right)\left(\frac{p_i}{\rho_i}\right)-\frac{1}{2}\left(\frac{p_i}{\rho_f}\right).$$

Equating these expressions, multiplying by $2\rho_f$ and rearranging gives the required result. **[5 marks]**

2. The motion of the material at any in the disc point is determined by the gravitational field at that point. If motion *perpendicular* to the plane does not take the star into regions where the gravitational forces *parallel* to the plane are significantly different from those acting when the star is in the plane, then the perpendicular motion will not significantly affect the motion in the plane. And *vice versa*; if the motion in the plane does not change the gravitational forces acting perpendicular to the plane, then this perpendicular motion will be independent of the motion in the plane. Because the *motions are relatively small* both the above assumptions are good. **[4 marks]**

We estimate how long it takes the star to travel the vertical scale-height

t=3.09x10^{19}/2x10^4=1.55x10^{15}s=1.55x10^{15}/3.16x10^7=4.91x10^7yr which is a long time from stellar dynamics in galaxies point of view. **[3 marks].**

The bulk motion around the centre is irrelevant because it is in-plane, not out of plane **[1 mark]**

3. If the circular velocity $\Theta(r)$ is constant: $\Theta(r)=\Theta_o=\text{constant}$, then the angular velocity $\Omega(r)$ is given by

$$\Omega(r)\equiv\frac{\Theta(r)}{r}=\frac{\Theta_o}{r}\propto\frac{1}{r}.$$ **[3 marks]**

4. From equation of the question, we have $A(r)-B(r)=\frac{\Theta(r)}{r}=\Omega(r)$,

where $\Omega(r)$ is the angular velocity. Putting in values, we get

$$\Theta(r_o)=r_o[A(r_o)-B(r_o)]=8.5\times[14.4+12.0]\ \text{km s}^{-1}=224.4\ \text{km s}^{-1}$$ **[2 mark]**

The period T_o is given by

$$T_o=\frac{2\pi}{\Omega(r_o)}.$$

From the equations, we have

$$T_o = \frac{2\pi}{\left[A(r_o) - B(r_o)\right]} = \frac{2\pi}{26.4\times10^3}\times 3.1\times10^{19}\ \text{s} = \frac{2\pi}{26.4\times10^3}\times\frac{3.1\times10^{19}}{3\times10^7}\ \text{y} \approx 2.4\times10^8\ \text{y}$$

[2 marks]

If $\Theta(r)\approx\Theta_o = \text{constant}$, then

$\frac{d}{dr}\Theta(r) = 0$ and $A(r)\approx -B(r)\approx \frac{1}{2}\frac{\Theta_o}{r}$.**[2 marks]**

[Total marks available 22]