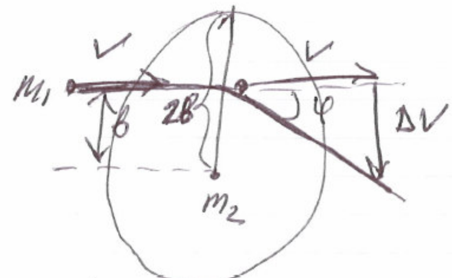
Physics of Galaxies
ANSWERS: SET NUMBER 5

1. and 2.

$$F = \frac{G m_1 m_2}{r^2} \qquad r \to b$$

$$F = \frac{G m_1 m_2}{b^2}$$



Impulse $I = F\Delta t$; $\Delta t = 2b/V$; $F\Delta t = m_1 \Delta V$

$$I = \frac{G m_1 m_2}{b^2}\,\frac{2b}{V} = \frac{2G m_1 m_2}{Vb} = m_1 \Delta V \Rightarrow$$

$$\Delta V = \frac{2G m_2}{bV}$$ drop subscript "2".

$$\tan\varphi = \frac{\Delta V}{V} = \frac{2Gm}{bV^2} \;;$$

For N number of stars $N = \mathbb{V}\cdot n$ where $\mathbb{V}$ is the total volume, n is the number density (number of stars per unit volume). Volume of influence of a star $V_{star} = \frac{\mathbb{V}}{N} = n^{-1}$, but $V_{star} \sim b^3$

$$b^3 \approx n^{-1} \Rightarrow \quad b \approx n^{-1/3} \Rightarrow$$

$$\tan\varphi = \frac{2G n^{1/3} m}{V^2}$$

[5 marks] and [5 marks]

3. Given the adiabatic law

$p = K\rho^{\gamma}$,

where K is a constant, then

$$u^2 = \left[\frac{\partial p}{\partial \rho}\right]_{\text{adiabatic}} = \frac{\partial}{\partial \rho}\left(K\rho^{\gamma}\right) = \gamma K \rho^{\gamma-1}.$$

Hence

$u = (\gamma K)^{1/2}\rho^{(\gamma-1)/2}$. **[3 marks]**

4. We have

$\rho = 2 n_{H_2} m_H$,

where n_{H_2} is the number-density of hydrogen molecules and m_{H} is the mass of the hydrogen atom. Hence

$$\tau_{\mathrm{ff}} \sim \left(2Gn_{\mathrm{H}_2}m_{\mathrm{H}}\right)^{-1/2}$$
$$= \left[2\times 6.67\times 10^{-11}\times 10^{10}\times 1.67\times 10^{-27}\right]^{-1/2}$$
$$\approx 2\times 10^{13}\ \mathrm{s} \approx 10^{6}\ \mathrm{y}.$$

[3 marks]

5. If

$$k < \left(\frac{4\pi G\rho}{u^2}\right)^{1/2},$$

then we have from the dispersion relation

$$\omega^2 < 0,$$

so that ω is imaginary. Hence writing

$$\omega = \pm i\alpha,$$

where α is a real positive number and substituting into the wave-like solution for the density perturbation $\rho_1(t,x)$

$$\rho_1(t,x) = \rho_{10}e^{i(\omega t - kx)}$$
$$= \rho_{10}e^{\mp\alpha t}e^{-ikx}$$

The second line of the latter equation shows that we have a standing wave with amplitude $\rho_{10}e^{\mp\alpha t}$. **[5 marks]**

[Total marks available 21]