Physics of Galaxies

ANSWERS: SET NUMBER 4

1.

$$M_{\text{total}} = \int_{M_l}^{M_h} \phi(M) M dM = \int_{M_l}^{M_h} \phi_* \left(\frac{M}{M_*}\right)^{-\alpha} M dM = \frac{\phi_* \left(M_h^{2-\alpha} - M_l^{2-\alpha}\right)}{M_*^{-\alpha}(2-\alpha)} =$$

$$\frac{\phi_* M_*^2}{(2-\alpha)} \left(\left(\frac{M_h}{M_*}\right)^{2-\alpha} - \left(\frac{M_l}{M_*}\right)^{2-\alpha} \right) = \frac{\phi_* M_*^2}{(\alpha-2)} \left(\frac{M_l}{M_*}\right)^{2-\alpha} \left(1 - \left(\frac{M_h}{M_l}\right)^{2-\alpha} \right) =$$

$$\frac{\phi_* M_*^2}{(\alpha-2)} \left(\frac{M_l}{M_*}\right)^{2-\alpha} \left(1 - \left(\frac{M_l}{M_h}\right)^{\alpha-2} \right) \approx \frac{\phi_* M_*^2}{(\alpha-2)} \left(\frac{M_l}{M_*}\right)^{2-\alpha}$$

In the last line we used the fact that from Salpeter IMF $\left(\frac{M_l}{M_h}\right) \approx 0.001 << 1$ and the fact that alpha =2.35>2 **[4 marks].** The total mass of the galaxy is prescribed by the low mass stars because Salpeter IMF shows that there are a lot more low mass stars than high mass stars in the distribution. [1 mark] The total luminosity is prescribed by the high mass stars because the mass-luminosity empirical relation, $L(M) = L_* (M / M_*)^{3.3}$, tells us that more massive stars are significantly more luminous (note the power of 3.3!) [1 mark] **[1+1=2 marks]**.

2. By the definition of flux-density *F*, we have for the luminosity *L* of a galaxy that is at distance *d*,

$$L = \left(4\pi d^2\right) \times F.$$

Using the expression for *d*, we get

$$L = 4\pi (zc / H_o)^2 F . \qquad \textbf{[4 marks]}$$

If a galaxy of size *R* is at a distance *d*, its angular diameter θ is given by

$$R = d \times \theta .$$

If we substitute for *R*, and set *k* = 1 (no marks lost for not setting *k* = 1), we get

$$M \sim (d \times \theta) \times \frac{v^2}{G} = \left(z \frac{c}{H_o} \right) \frac{v^2}{G} \theta \quad \textbf{[3 marks]}$$

Then we have directly

$$\frac{M}{L} \sim \frac{\left(z \frac{c}{H_o} \right) \frac{v^2}{G} \theta}{4\pi \left(z \frac{c}{H_o} \right)^2 F} = \left[\frac{1}{4\pi} \frac{v^2 \theta}{Gc \times zF} \right] H_o \propto H_o \propto h.$$

All the quantities in the square bracket are either constants or are directly observed. The measured mass-luminosity ratio is therefore directly proportional to the Hubble constant. **[4 marks]**

3. The mean-free path of stars – before they suffer significant deflection from their original paths – in most parts of a galaxy is many times the size of the observable universe; the corresponding mean-free time is orders of magnitude more than the age of the universe. Only in the central regions of galaxies, and in globular clusters, are these quantities comparable with galactic sizes and ages. Most stars can therefore be considered to move independently of other individual stars. **[5 marks]** A *strong*

gravitational interaction is one in which a star is deflected through a significant angle, e.g. one radian or e.g. 45 degrees. **[2 marks]**

4. Using the given values, we have, in MKS units, $G = 6.7 \times 10^{-11}$, $m = 2 \times 10^{30}$, $V = 4 \times 10^{4}$, $n = 0.1 \times (3.3 \times 10^{16})^{-3}$. The angle φ then becomes in radians

$$\frac{2 \times 6.7 \times 10^{-11} \times 2 \times 10^{30} \times 0.1^{1/3}}{(4 \times 10^{4})^{2} \times 3.3 \times 10^{16}} = 2.55 \times 10^{-6}$$

The number of arcseconds in a radian is $(180/\pi) \times 60 \times 60 = 2.06 \times 10^{5}$

so $\varphi = 2.55 \times 10^{-6} \times 2 \times 10^{5} = 0.5$ arcsec **[3 marks]**

[Total marks available 27]