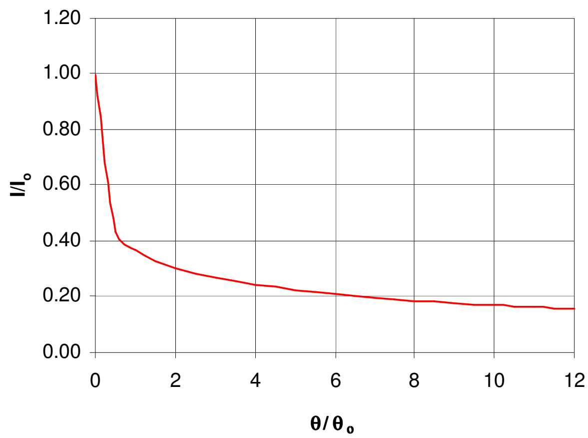
Physics of Galaxies

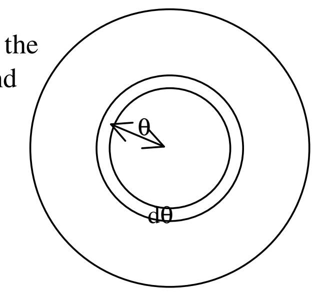
ANSWERS: SET NUMBER 3

1. The surface brightness, I, of a galaxy is its flux density, dF, per unit solid angle, $d\Omega$, as a function of position (projected angle) in the galaxian image, i.e. $I(\theta) = \dfrac{dF}{d\Omega}$ **[3 marks]**



Note for marker: look for following in the answer: at abscissa of 0 the function value should be 1 [1 mark], at abscissa of 1 the function value show be 1/e=0.37 [1 mark], [1 mark] for showing very slow decaying tail for large abscissa values. **[3 marks]**

Because $I(\theta)$ is given only as a function of θ, we can assume that the image of the galaxy has circular symmetry. Consider an annulus of (angular) radius θ and width $d\theta$. The solid angle $d\Omega$ subtended by this annulus is given by



$$d\Omega = 2\pi\theta d\theta$$

and the flux dF coming from it is given by

$$dF = I(\theta)2\pi\theta d\theta \,.$$

The total flux density of the galaxy is therefore given by

$$F = \int_0^\infty I(\theta)2\pi\theta d\theta \,.$$ **[3 marks]**

Substituting for the de Vaucouleurs profile, we get

$$F = \int_0^\infty I(0)\exp\left[-\left(\frac{\theta}{\theta_o}\right)^{1/4}\right]2\pi\theta d\theta = 2\pi I(0)\theta_o^2\int_0^\infty \exp\left[-\left(\frac{\theta}{\theta_o}\right)^{1/4}\right]\left(\frac{\theta}{\theta_o}\right)d\left(\frac{\theta}{\theta_o}\right).$$

If we put $x = \left(\dfrac{\theta}{\theta_o}\right)^{1/4}$,

Then

$$F = 2\pi I(0)\theta_o^2\int_0^\infty e^{-x}x^4 d\left(x^4\right) = 2\pi I(0)\theta_o^2 \times 4\int_0^\infty e^{-x}x^7\,dx = \pi I(0)\theta_o^2 \times 8 \times 7!$$

using the standard integral given in the question. Hence, $F = 8!\pi\theta_o^2 I(0)$ **[5 marks]**

2. Putting in $\theta = \theta_e$ in the equation

$$I(\theta) = I(0)\exp\left[-7.67\left(\frac{\theta}{\theta_e}\right)^{1/4}\right]$$ we get $I(\theta_e) = I_e = I(0)\exp[-7.67]$

option 1

using the latter equation (expressing I(0) by means of I_e)

$$I(\theta) = I_e\exp[7.67]\exp\left[-7.67\left(\frac{\theta}{\theta_e}\right)^{1/4}\right] = I_e\exp\left[-7.67\left[\left(\frac{\theta}{\theta_e}\right)^{1/4} - 1\right]\right].$$

option 2 (more convoluted, but instructive in relating powers of 10 and *e*)

Then use identities $10^{-x} = e^{\ln(10^{-x})} = e^{-x\ln(10)}$ and

$$10^{-3.33} = e^{\ln(10^{-3.33})} = e^{-3.33\ln(10)} = e^{-3.33x2.30} = e^{-7.67}$$ and

$$10^{-3.33\left(\frac{\theta}{\theta_e}\right)^{1/4}} = e^{-7.67\left(\frac{\theta}{\theta_e}\right)^{1/4}}$$

Thus,

$$I(\theta_e) = I_e = I(0)\exp[-7.67] = I(0)10^{-3.33}$$

$I(0) = I_e 10^{3.33}$.Therefore,

$$I(\theta) = I_e 10^{3.33}\exp\left[-7.67\left(\frac{\theta}{\theta_e}\right)^{1/4}\right] = I_e 10^{3.33} 10^{\left[-3.33\left(\frac{\theta}{\theta_e}\right)^{1/4}\right]} =$$

$$= I_e 10^{\left[-3.33\left\{\left(\frac{\theta}{\theta_e}\right)^{1/4} - 1\right\}\right]} = I_e\exp\left[-7.67\left[\left(\frac{\theta}{\theta_e}\right)^{1/4} - 1\right]\right]$$

[6 marks] **[Total Set 3 marks available 20]**