MTH6132, Relativity

Problem Set 5

Due 14th November 2018

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1. Starting with the covariant derivative of a contravariant vector and assuming that the covariant derivatives obey the usual Leibniz rule for differentiation of products and that the covariant derivative of a scalar is the partial derivative, prove that the covariant derivative of a covariant vector V_a is given by:

$$\nabla_b V_a = \partial_b V_a - \Gamma^c{}_{ab} V_c$$

Hint: Let the scalar $\Phi = V_a W^a$ and recall that $\nabla_b \Phi = \partial_b \Phi$ for scalars. Expand and substitute for $\partial_b W^a$ from the definition of $\nabla_b W^a$.

2. A type (0,2) tensor is *conserved* if

$$\nabla^a T_{ab} = 0.$$

Show that if X^a satisfies the equation $\nabla_{(a} X_{b)} = 0$, and T_{ab} is symmetric, then the vector $V_a = T_{ab} X^b$ satisfies

$$\nabla^a V_a = 0.$$

3. Let $S_b{}^c$ denote a $(1, 1)$ tensor.

(a) Give the formula for the covariant derivative $\nabla_a S_b{}^c$ in terms of the connection coefficients.

(b) Show that $\nabla_a \delta_b{}^c = 0$.

(c) Show that if the dimension of the manifold is 4, then $\delta_a{}^a = 4$.

4. Show that changing the geodesic parameter λ to σ in such a way that $\sigma = \sigma(\lambda)$, the geodesic equation only keeps its form in σ if $\sigma = a\lambda + b$.

Please use Reading Week to fill your knowledge gaps, catch up on exercises from the Problem Sets, revise your lecture notes *and* the printed notes (checking all the calculations), and formulating some questions to ask me during Tutorials and Office Hours.