MTH 6132: (MATHEMATICAL) RELATIVITY

Lecturer: Professor Shahn Majid

Lectures: Monday 11am-12pm (Bancroft:113a), Thu 3pm-5pm (PP2)

Tutorials: Friday 9am-10am (Bancroft: 4.04)

Module webpage: All lecture notes, problem sheets, solutions and additional information will be made available on the modules QMplus page, found via http://qmplus.qmul.ac.uk/.

Syllabus: The module is split approximately into three parts.

- I Pre-relativistic Physics and Special Relativity (SR): Newtonian mechanics and Galilean Relativity. Maxwell equations and Special Relativity. Lorentz transformations and Minkowski spacetime. Clocks and rods in relative motion. Vectors in Special Relativity: 4-vectors, the Lorentz transformation matrix, 4-velocity, 4-momentum, 4-acceleration. Relativistic dynamics and collisions. Optics: redshift and aberration.
- II Calculus on Manifolds: Conceptual basis of General Relativity and the Equivalence Principle. Differential Geometry and tensor calculus; covariant derivatives and connections. Parallel transport, geodesics and the geodesic equation. Riemannian and Lorentzian geometry, the metric and the curvature of spacetime.
- III General Relativity (GR), Theory & Applications: Einstein field equations. Newtonian (weak field) limit. Schwarzschild solution. Tests of General Relativity: precession of perihelion and the bending of light. Black holes and gravitational collapse. If time allows, other topics will be explored (e.g., Cosmology, Gravitational Waves, Penrose diagrams, related open problems in Geometric Analysis).

Assessment: The total mark for this course will be based on a 100% final examination in January 2020. There will be a Revision Lecture before the final examination period. Details will be announced closer to the relevant dates.

Feedback, Coursework and Quizzes: Your feedback will be in many forms.

- \- In tutorial, we will combine learned concepts through fortnightly Problem Sets; myself and a TA will be available to give you feedback on your approach to these problems and answer questions. You should submit solutions to the * problems for marking at the deadlines shown on the qmplus page for each sheet and I will then post solutions. Tutorials are weekly and it is essential that you attend both weeks as the problem sets are too long to cover in one tutorial. There is no tutorial in Week 1 but you should start work on Problem set 1 by yourself and ask me any questions in lectures, then finish off in the Week 2 tutorial.

- \- You will receive feedback on your writeups both for mathematical content and clarity of exposition.
- *Although these will not contribute to your final mark, attending lectures, participating fully during tutorials, and completing Problem Sets (not just the submitted exercises) are all vital to mastering the content of this module!*

Office Hours: You are welcome to discuss questions and get further feedback during the tutorials — these are not limited to discussion of courseworks. In addition I will usually have an office hour once a week in G25 in the maths building, please check my departmental page http://www.qmul.ac.uk/maths/profiles/majids.html for the latest information.

Engagement Policy: Engagement will be monitored in line with the School's Student Engagement Policy, including attendance as well as collected work. *Please also allow me to emphasize that lectures, tutorials, Problem Set submissions and quizzes are not optional*.

Learning outcomes: At the end of the course you should be able to:

1. Explain the principles of special relativity and the key steps leading to the Lorentz transformations.
2. Employ a geometrical approach to study SR effects by using Minkowski geometry and spacetime diagrams.
3. Understand and use 4-vectors in a variety of different settings relevant to relativistic dynamics and collisions.
4. Use the techniques of tensor algebra and tensor calculus on (curved) manifolds.
5. Understand covariant derivatives, connections, parallel transport, geodesics and curvature on manifolds.
6. Appreciate the importance of the metric tensor and the significance and applications of the geodesic equations.
7. Explain the terms comprising Einstein's equations and understand the Newtonian (weak field) limit of the theory.
8. Understand and explain the applications of the General Relativity, including the Schwarzschild solution, experimental tests of the theory, black holes and gravitational collapse.

References: People often speak of the beauty of the General Theory of Relativity. However, different people are drawn to it for different reasons. Pure mathematicians may be mostly attracted to it for the mathematical elegance of the theory, whereas physicists are typically more interested in understanding the physical concepts of the theory. Others may focus on the richness of the field equations and their solutions, or developing observable tests and applications of the theory. Indeed, Einsteins own views changed during his lifetime. When he first developed the theory, he placed great emphasis on physical ideas and thought experiments. Later on, he was drawn to more geometrical ideas and spent much of the second half of his life attempting to develop geometric modifications and extensions of his theory. There is no shortage of books on both the special and general theories of relativity, at varying levels and with differing emphasis on the mathematical or physical aspects of the theories. The campus library has a good stock of books, mostly located in Section QC6 of the main building (on the first floor).

Aside from the course printed notes (see below), you may find some of the following references useful in your study of Relativity:

- \- R. D'Inverno, *Introducing Einsteins Relativity*, Clarendon Press, Oxford (1992). This book is self-contained and most of the contents of our lectures are covered, although with a slightly different approach occassionally. D'Inverno explains the physics behind the mathematics of GR and develops the machinery of tensor calculus carefully. The campus library has 7 copies of this book, although they are restricted to one week loan.
- \- B. F. Schutz, *A First Course in General Relativity*, Cambridge University Press. New edition (2009). This is more physical in its approach (i.e. there are fewer equations). Both the special and general theories are covered although the emphasis is on the latter.
- \- J. B. Hartle, *Gravity: An Introduction to Einsteins General Relativity*, Addison-Wesley. This is a good "Physics first" introduction to Relativity, being quite intuitive and providing discussion on applications of the theory. It has a long list of exercises, and also a discussion of the underlying Mathematics.
- \- L. Ryder, *Introduction to General Relativity*, Cambridge University Press. New edition (2009). Both the special and general theories are covered although the emphasis is on the latter. Covers a lot of useful material, including some more advanced than those discussed in Lectures.
- \- W. Rindler, *Relativity: Special, General, Cosmological*, Oxford University Press. New edition (2006). Very complete presentation discussing many of the topics covered in the lecture.
- \- N. M. J. Woodhouse, *General Relativity*, Springer Undergraduate Mathematics Series (2007). Lecture notes drawn from a course given at Oxford by the author to final-year mathematics students, very much in the tradition of physical applied mathematics.

- \- R. L. Faber, *Differential Geometry and Relativity Theory*, Marcel Dekker, Inc. (1983). A beautiful book for those who wish to study Relativity starting with classical Euclidean differential geometry of curves and surfaces.
- \- Y. Choquet-Bruhat, *Introduction to General Relativity, Black Holes & Cosmology*, Oxford University Press (2015). Written by Yvonne Choquet-Bruhat, one of the (past and current) leaders in the field, this text provides a comprehensive account of modern Mathematical Relativity with material suitable for both undergraduate (Chapters 1-7) and graduate study (Chapters 8-10).